On approximations of the wave drift forces acting on semi-submersible platforms with heave plates

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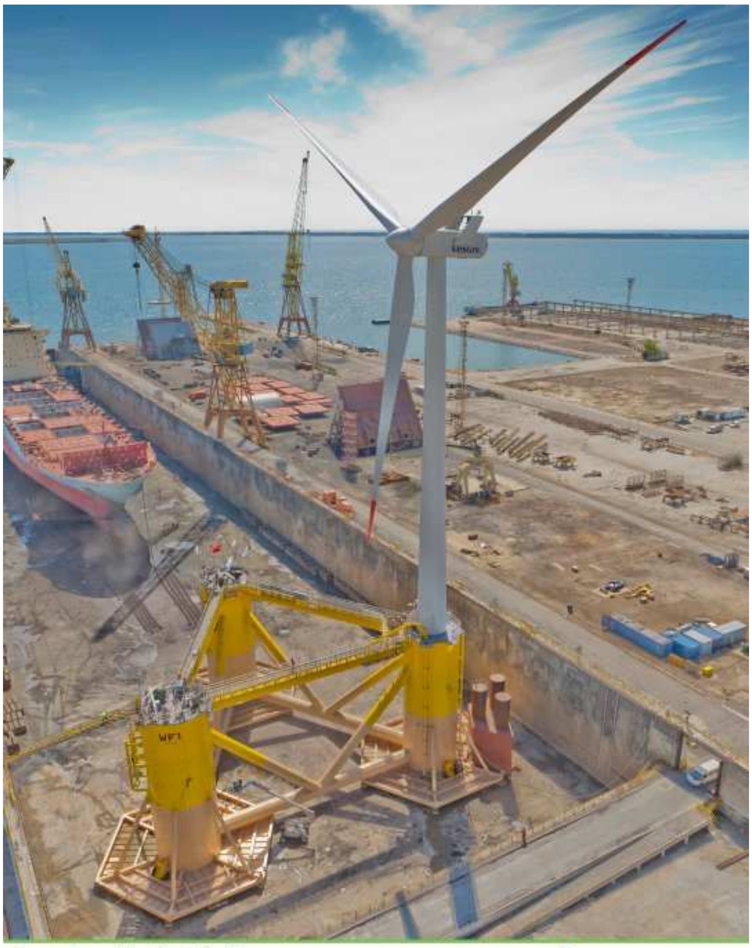
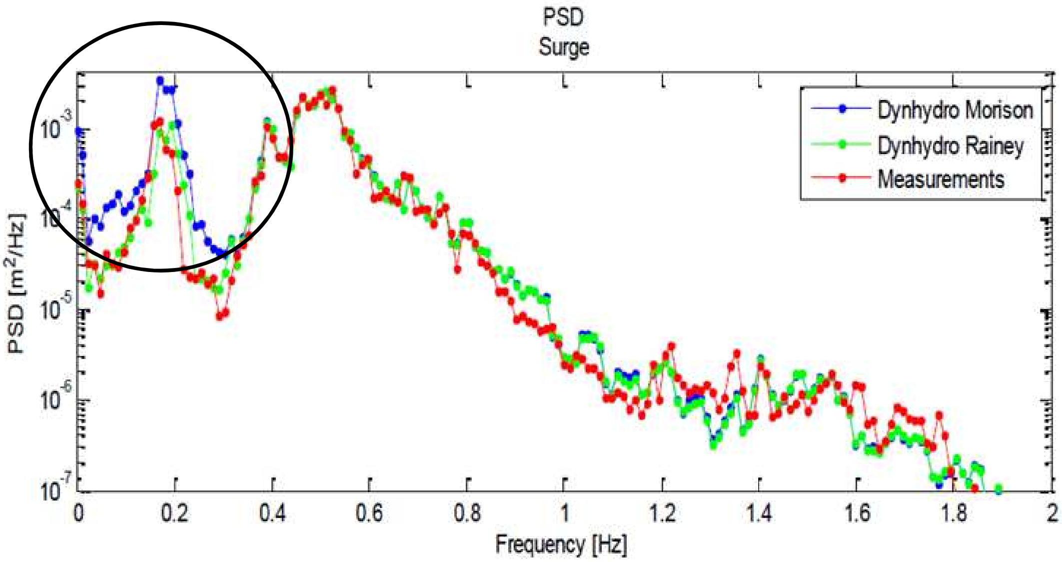


Aguçadoura WindFloat Prototype
September 2011- MPG/Lisnave Workshop, Portugal

PRINCIPLE
renewable energy delivered

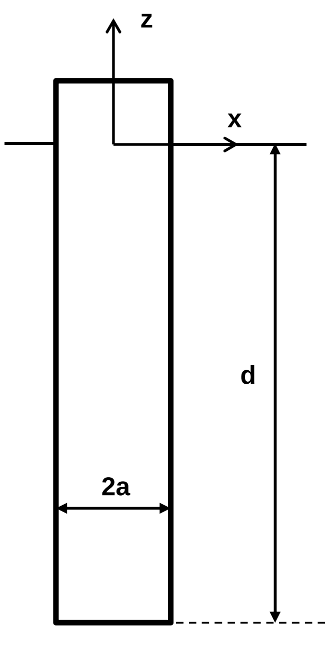
Tests in irregular waves. No wind.
Surge power spectrum.



(from Lacaze's Ph.D. thesis, 2015)

Single vertical column in heave only

1. Solve heave motion equation through Hooft's method:



Incoming wave potential

$$\Phi_I(x,z,t) = \frac{A\,g}{\omega}\,\mathrm{e}^{kz}\,\sin(kx-\omega t)$$

Vertical load at bottom

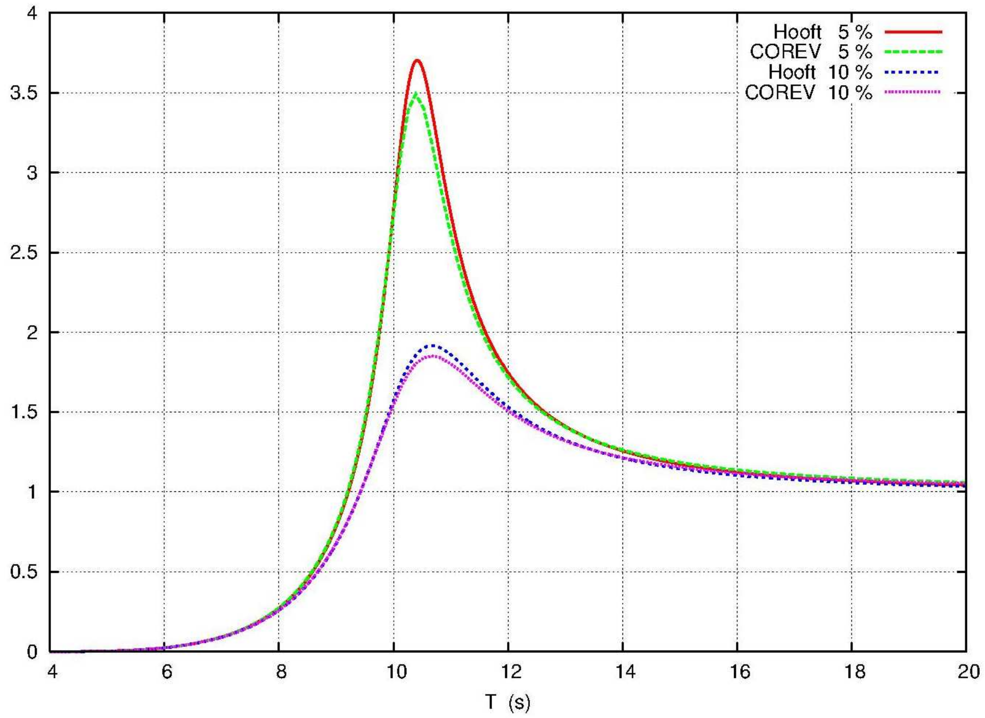
$$F_z(t) = \rho\,g\,A\,(\pi a^2 - 2ka^3)\,\mathrm{e}^{-kd}\,\cos\omega t$$

Heave response

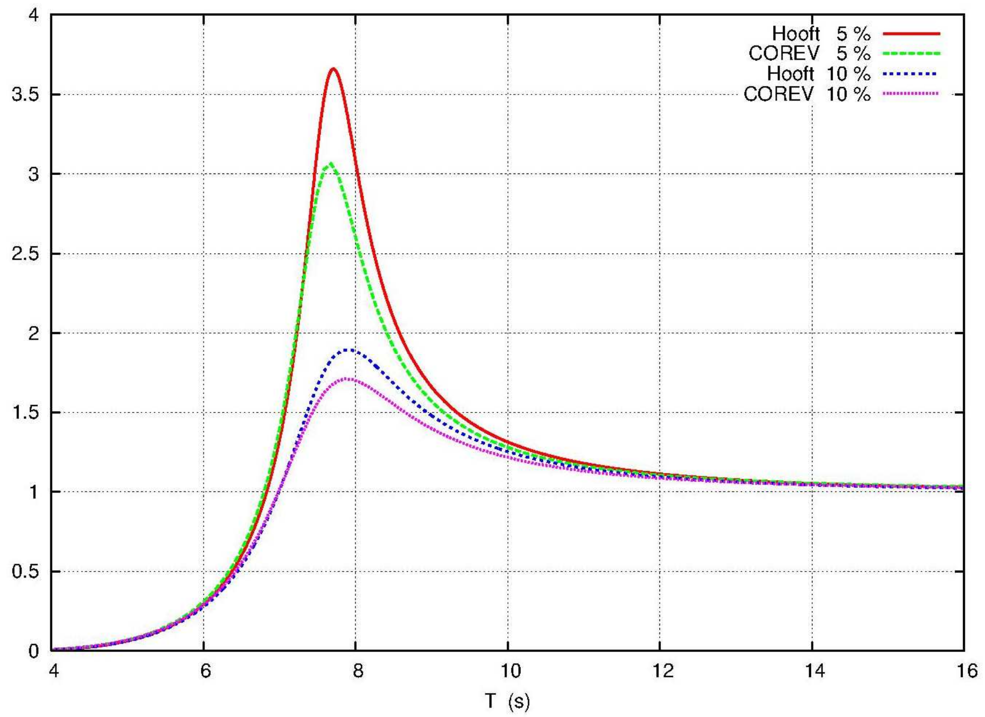
$$Z(t) = A\,\Re\left\{z_0\,\mathrm{e}^{-\mathrm{i}\,\omega t}\right\}$$

with z_0 the complex RAO

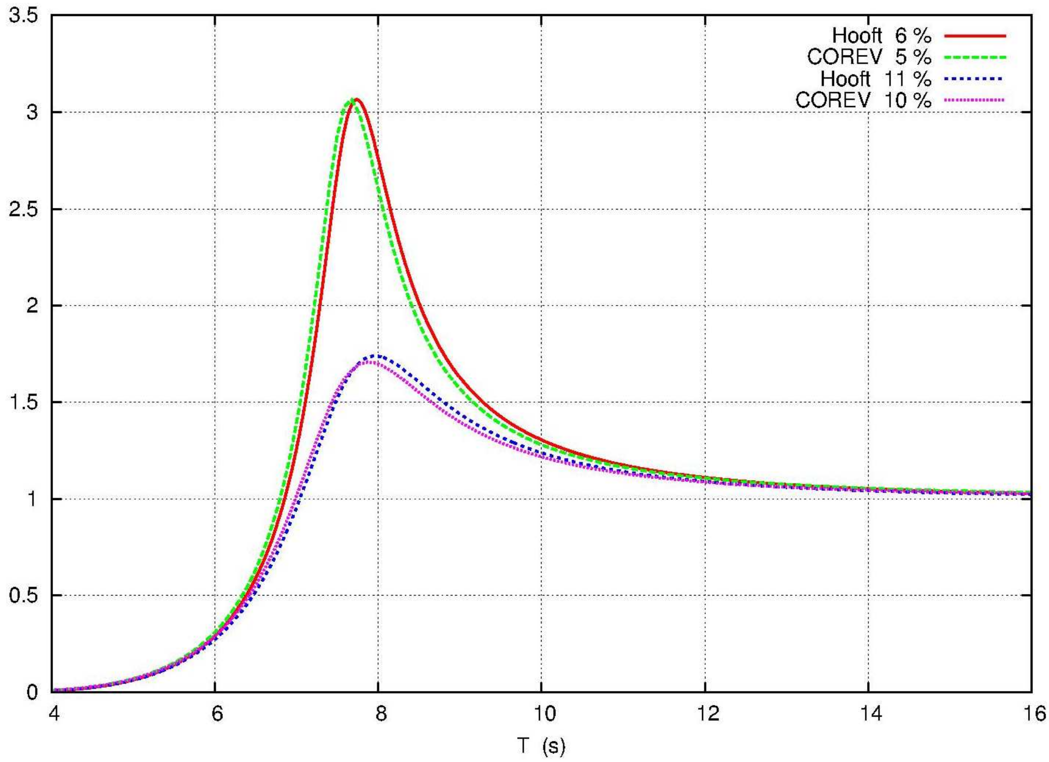
$$z_0 = \frac{\rho\,g\,(\pi\,a^2 - 2k\,a^3)\,\mathrm{e}^{-kd}}{-\omega^2(M+M_a) - \mathrm{i}\,\zeta\,\omega\,B_C + K}$$



Heave RAO. Draft 24 m. Radius 4 m.



Heave RAO. Draft 12 m. Radius 4 m.



Heave RAO. Draft 12 m. Radius 4 m.

Drift force from the inertia term in the Morison equation

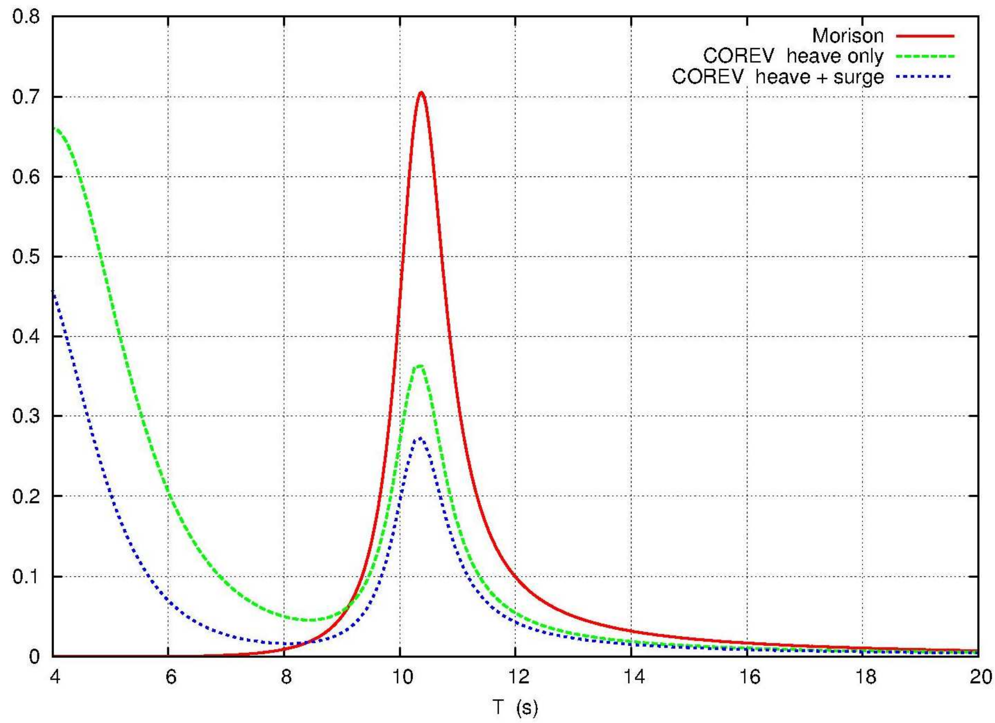
$$\mathrm{d}F_x = 2\,\rho\,\pi\,a^2\,\Phi_{xt}(0,z,t)\;\mathrm{d}z$$

to be integrated from $-d + Z(t)$ up to $\eta(0,t)$.

The non zero time averaged value results from the integration between $-d + Z(t)$ and $-d$, leading to the second-order drift force:

$$F_{dx\,\mathrm{Morison}} = \rho\,g\,\pi\,A^2\,k\,a^2\,\mathrm{e}^{-kd}\;\Im\,\{z_0\}$$

compared with "exact" calculations with COREV code.



Horizontal drift force Fd / rho g a A^2.
Draft 24 m. Damping 5 %.

Introducing Rainey's equations

J. Fluid Mech. (1989), *vol.* 204, *pp.* 295–324 295
Printed in Great Britain

A new equation for calculating wave loads on offshore structures

By R. C. T. RAINEY
WS Atkins Engineering Sciences, Woodcote Grove, Epsom, Surrey KT18 5BW, UK

Slender-body expressions for the wave load on offshore structures

By R. C. T. Rainey

Proc. Royal Soc. A, 1995

(a) *Force per unit immersed length*

If the cross-sectional area of a structural member is c and its 2D added mass is written as the 3D matrix $\boldsymbol{\mathsf{M}}$ (by defining $\boldsymbol{\mathsf{M}}\boldsymbol{x} = 0$ whenever the vector $\boldsymbol{x}$ is axial – throughout this paper bold sans-serif capitals denote matrices, and bold letters denote vectors), then the force per unit length on it is:

$$\rho c[\boldsymbol{a}-\boldsymbol{g}]_{\mathrm{T}}+\boldsymbol{\mathsf{M}}[\boldsymbol{a}+(\boldsymbol{l}\cdot\boldsymbol{\mathsf{V}}\boldsymbol{l})\boldsymbol{w}]-\boldsymbol{\mathsf{M}}\dot{\boldsymbol{u}}-2\boldsymbol{\mathsf{M}}\boldsymbol{\Omega}\,\boldsymbol{w}_{\mathrm{A}}+[(\boldsymbol{\mathsf{V}}+\boldsymbol{\Omega})\boldsymbol{\mathsf{M}}\boldsymbol{w}]_{\mathrm{T}}-\boldsymbol{\mathsf{M}}(\boldsymbol{\mathsf{V}}+\boldsymbol{\Omega})\boldsymbol{w}_{\mathrm{T}}. \quad (1)$$

(c) *Point loads at joints*

At a joint, each structural member terminating there produces a point load:

$$(\tfrac{1}{2}\boldsymbol{w}\cdot\boldsymbol{\mathsf{M}}\boldsymbol{w} - cp)\boldsymbol{l} - (\boldsymbol{l}\cdot\boldsymbol{w})\boldsymbol{\mathsf{M}}\boldsymbol{w} \quad (3)$$

where p is the pressure in the incident wave (i.e. $\nabla p = -\rho(\boldsymbol{a}-\boldsymbol{g})$), and the sense of the unit axial vector $\boldsymbol{l}$ is taken as outwards from the member end. This is exactly

Missile launching from a submarine

Kinetic energy in the fluid

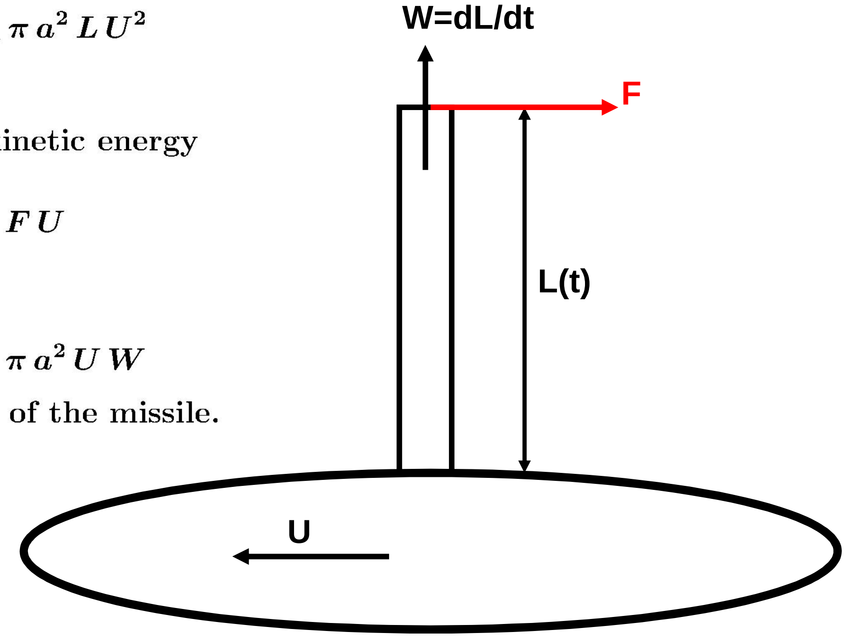
$$E_C = \frac{1}{2}\,\rho\, C_m\, \pi\, a^2\, L\, U^2$$

Variation of the kinetic energy

$$\frac{\mathrm{d}E_C}{\mathrm{d}t} = F\,U$$

$$\rightarrow F = \rho\, C_m\, \pi\, a^2\, U\, W$$

applied at the tip of the missile.



Rainey's end load (horizontal component) :

$$F_{xe} = \rho\,\pi\,a^2\,U\,\left(W - \dot{Z}\right)$$

with $U = \Phi_x(0, -d, t)$, $W = \Phi_z(0, -d, t)$.

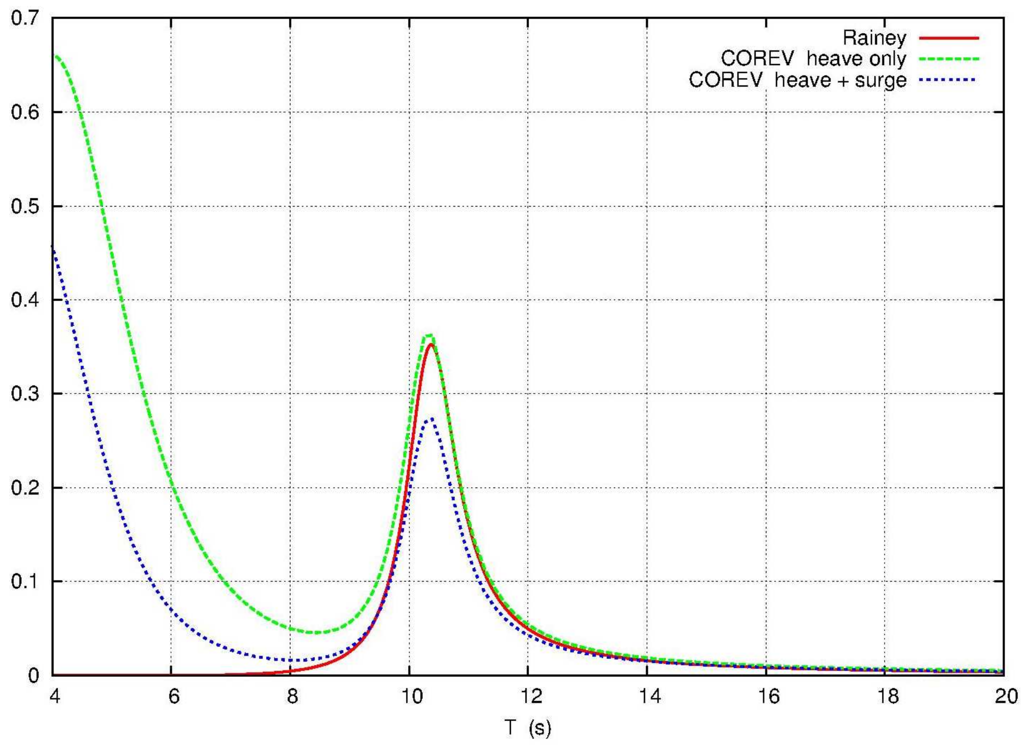
$\rightarrow$ time-averaged value

$$\overline{F_{xe}} = -\frac{1}{2}\,\rho\,g\,\pi\,A^2\,k\,a^2\,\mathrm{e}^{-kd}\,\Im\{z_0\}$$

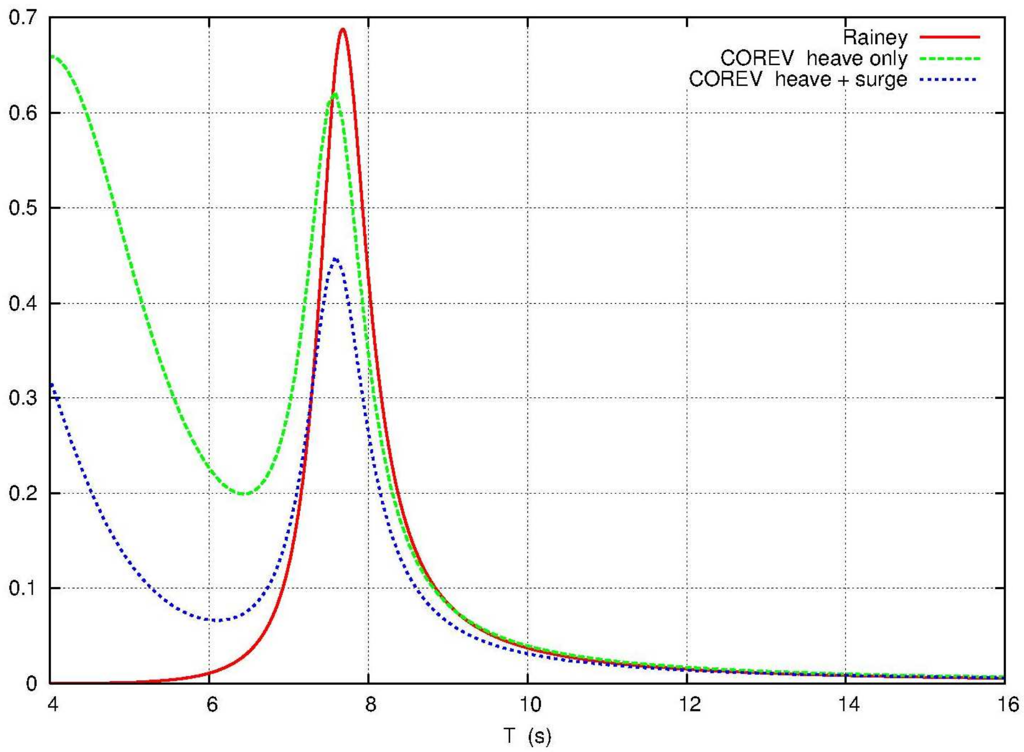
so that the drift force becomes

$$F_{dx\,\mathrm{Rainey}} = \frac{1}{2}\,\rho\,g\,\pi\,A^2\,k\,a^2\,\mathrm{e}^{-kd}\,\Im\{z_0\}$$

half the Morison value.



Horizontal drift force. Draft 24 m. Damping 5%.



Horizontal drift force. Draft 12 m. Damping 5%.

Drift force from far-field waves (Maruo 1960)

$$F_{dx} = \rho\, g\, A^2\, \frac{1}{k} \left[\frac{1}{2\pi} \int_0^{2\pi} H(\theta)\, H^*(\theta)\, \cos\theta\, \mathrm{d}\theta + \Re(H(0)) \right]$$

with $H(\theta)$ the Kochin function.

The heaving column creates a flux $Q = -\pi\, a^2\, \dot{Z}(t)$ at its foot which, owing to its deep draft and small diameter, can be viewed, from the far field, as a submerged pulsating source. From Wehausen & Laitone, eq. (13.17"'), the resulting velocity potential is

$$\Phi_R = -\Re \left\{ \frac{\pi}{2}\, k\, a^2\, A\, z_0\, \omega\, \mathrm{e}^{k(z-d)}\; H_0(kR)\; \mathrm{e}^{-\mathrm{i}\,\omega t} \right\}$$

with H_0 the Hankel function.

The Kochin function is then

$$H(\theta) = -\mathrm{i}\, \frac{\pi}{2}\, k^2 a^2\, z_0\, e^{-kd}$$

and the drift force is

$$F_{dx} = \frac{1}{2}\, \rho\, g\, \pi\, A^2\, k\, a^2\, \mathrm{e}^{-kd}\; \Im\, \{z_0\}$$

in agreement with Rainey!

Vertical column with heave plate

Heave plate radius: 9 m.

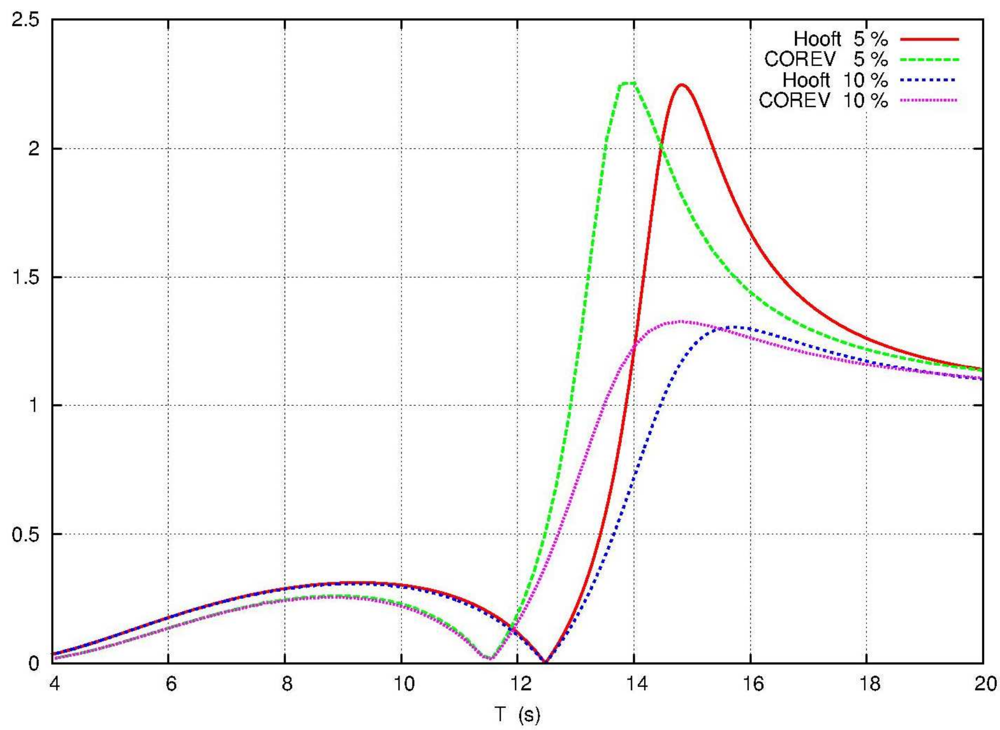
Thickness: 0.5 m.

Draft: 12 m.

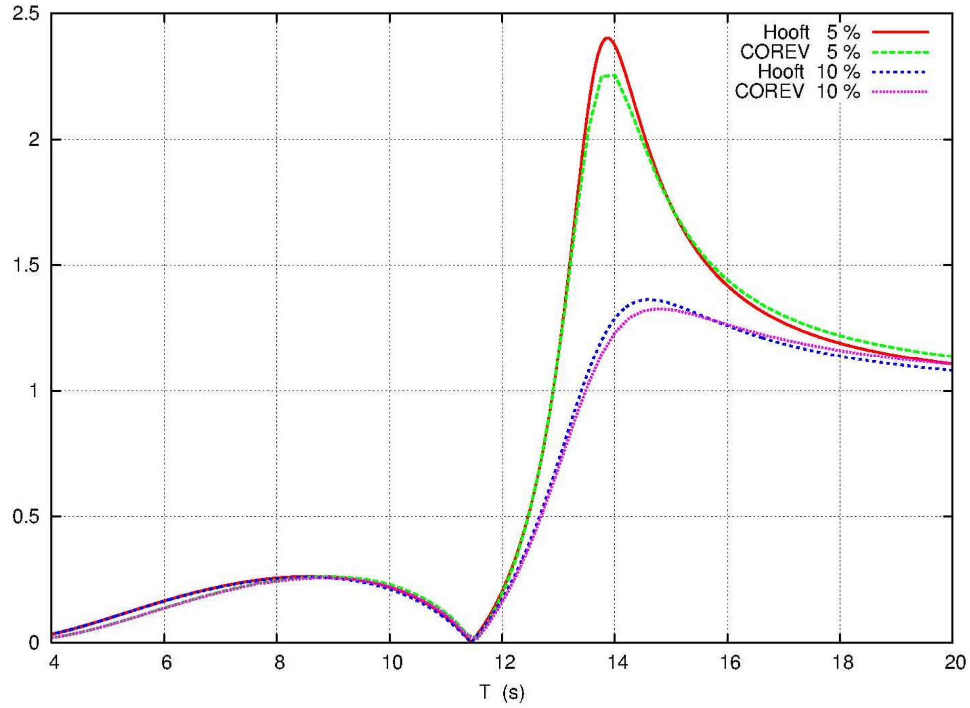
Added mass taken equal to $8/3\,\rho\,b^3$

Vertical exciting force:

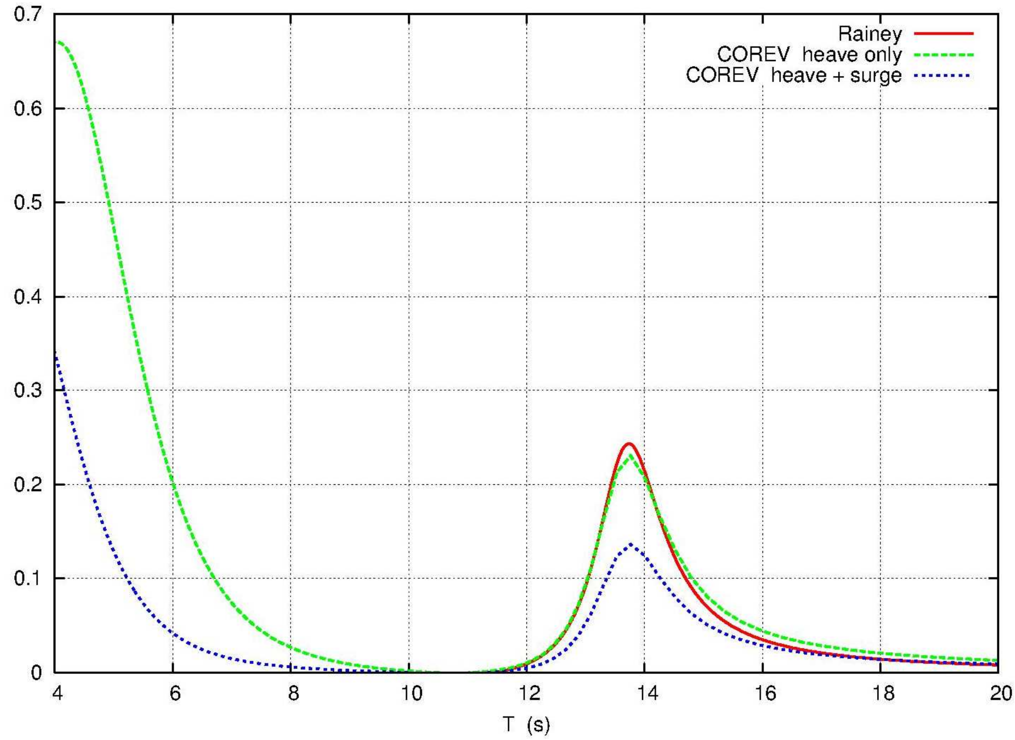
$$F_{ze} = \rho\, g\, A\, \left(\pi a^2 - \frac{8}{3}\, k b^3\right)\, \mathrm{e}^{-kd}\, \cos\omega t$$



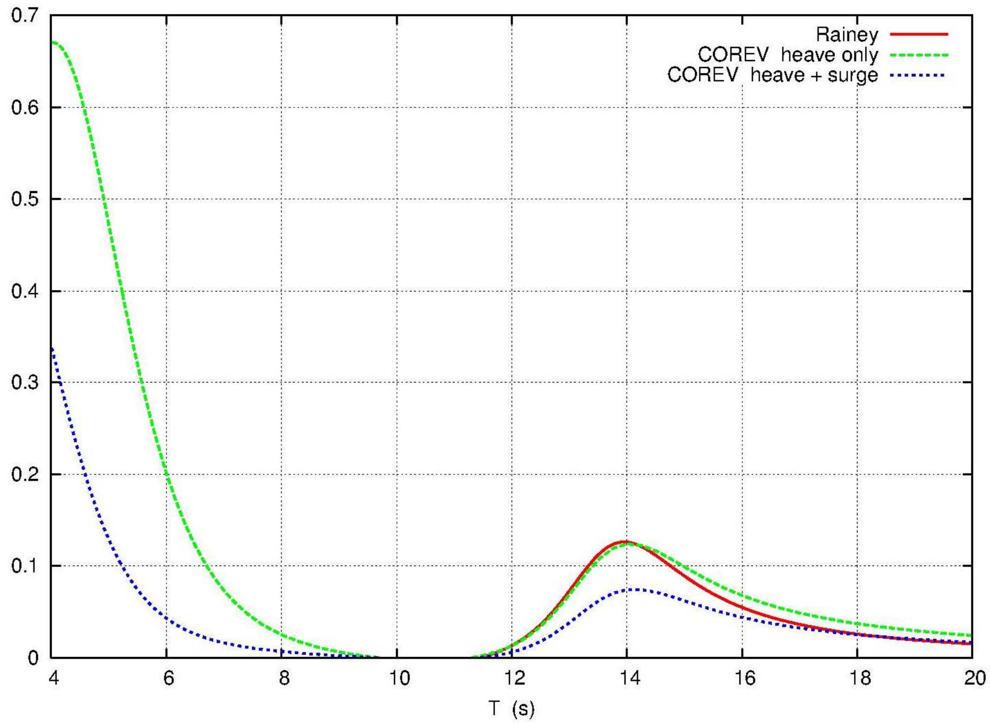
Vertical column with heave plate. Heave RAO.



Heave plate radius decreased to 8.5 m in Hooft. Heave RAO.



Horizontal drift force. Damping 5 %.



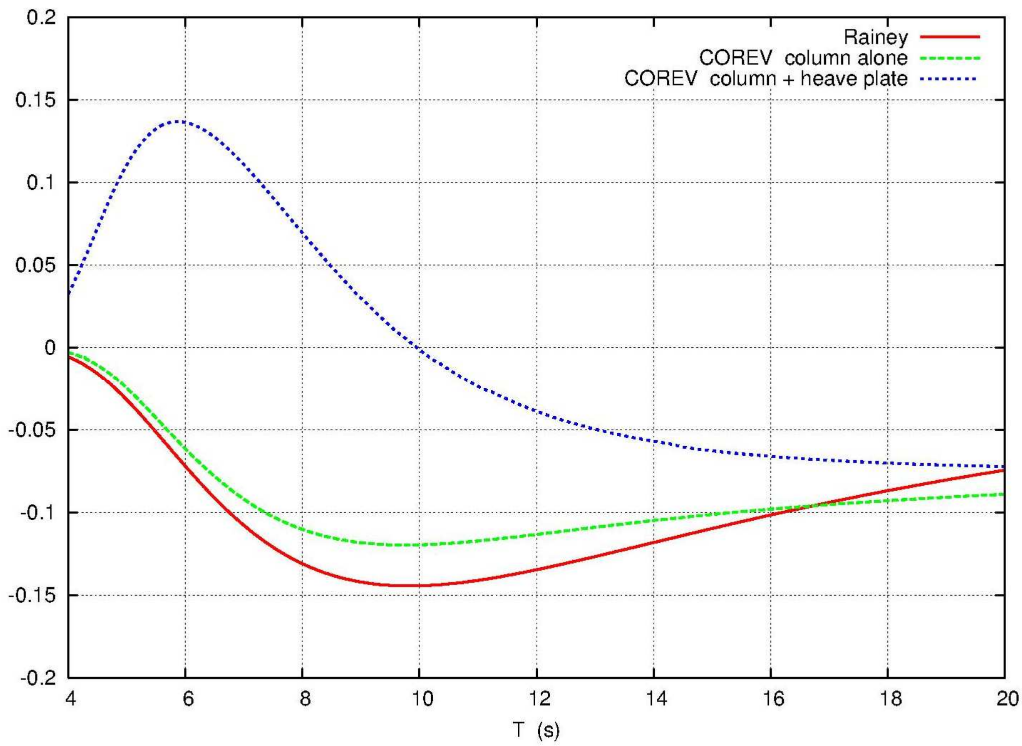
Horizontal drift force. Damping 10 %.

Drift moment in roll/pitch

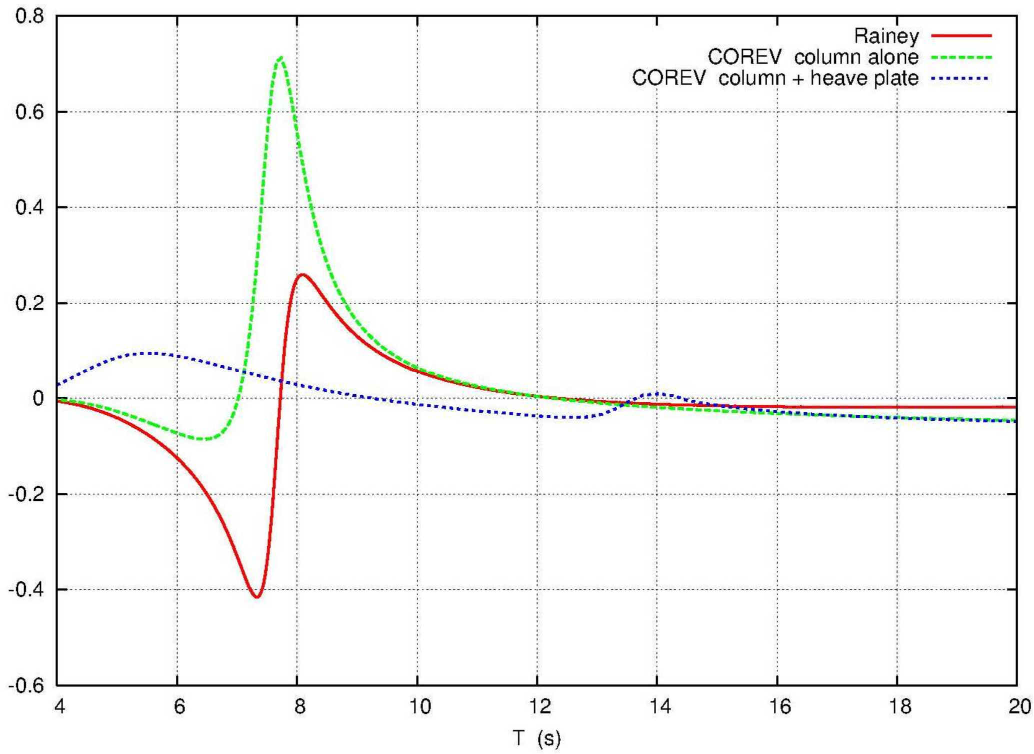
- Rainey / Morison predict that the drift force applies at the foot of the column (or column plus heave plate)
- Confirmed by COREV

Vertical drift force

- For the column alone should be easier to predict through Morison/Rainey approaches since free surface effects are not involved.
- The other way around: very poorly predicted!!!
- How to account for the heave plate?



Vertical drift force. Fixed.



Vertical drift force. Heave only. Damping 5 %.

Thank you for your attention. Questions?